MECHANICS





Original article

UDC 539.3

https://doi.org/10.23947/2687-1653-2022-22-1-4-13

Vibration analysis of a composite magnetoelectroelastic bimorph depending on the volume fractions of its components based on applied theory

Arkadiy N. Soloviev^{1,2,} Do Thanh Binh¹ D, Valerii A. Chebanenko³ D, Olga N. Lesnyak¹ D



Evgeniya V. Kirillova⁴

¹Don State Technical University (Rostov-on-Don, Russian Federation)

⊠ solovievarc@gmail.com

Introduction. Transverse vibrations of a bimorph consisting of two piezomagnetoelectric layers and located in the alternating magnetic field are investigated. Piezomagnetoelectric layers are multilayer composites with alternating piezoelectric and piezomagnetic layers. The mechanical and physical properties of such a composite are given by known effective constants.

Materials and Methods. The applied theory of multilayer plate vibrations takes into account the nonlinear distribution of electric and magnetic potential in piezoactive layers in the longitudinal and transverse directions. On the basis of this theory, the stress-strain state, the dependences of deflection, electric and magnetic potentials on the volume ratio of the composition of the hinged bimorph, are investigated. The electric potential is assumed to be zero at all electrodes, while the magnetic potential is zero at the inner boundary and unknown at the outer boundaries. Therefore, the distribution of electric and magnetic potentials in the middle of the layer are unknown functions. In the case of the magnetic potential, the distribution at the outer boundary is also unknown. In the problem, the Kirchhoff hypotheses for mechanical characteristics were accepted. The use of the variational principle and the quadratic dependence of the electric and magnetic potentials on the thickness of piezoactive layers made it possible to obtain a system of differential equations and boundary conditions.

Results. When the volume ratio of the composition of piezoactive bimorph materials changes, the electric potential in the middle of the layer changes nonlinearly. The magnetic potential in the middle of the layer and at the outer boundary increases almost linearly with an increase in the volume percentage of $BaTiO_3$. The dependence of the deflection in the middle of the layer is determined.

Discussion and Conclusions. An applied theory for calculating transverse vibrations of a bimorph with two piezomagnetoelectric layers is constructed. The dependence of the characteristics of the stress-strain state, electric and magnetic fields on the volume fractions of piezomagnetic and piezoelectric materials, is investigated.

Keywords: piezoelectrics, piezomagnetics, composite, bimorph, magnetoelectroelasticity, bending vibrations.

For citation: Soloviev, A. N., Do Thanh Binh, Chebanenko, V. A., Lesnyak, O. N., Kirillova, E. V. Vibration analysis of a composite magnetoelectroelastic bimorph depending on the volume fractions of its components based on applied theory. Advanced Engineering Research, 2022, vol. 22, no. 1, pp. 4–13. (In Russ). https://doi.org/10.23947/2687-1653-2022-22-1-4-13

²Southern Federal University (Rostov-on-Don, Russian Federation)

³Federal Research Centre the Southern Scientific Centre of the Russian Academy of Sciences (Rostov-on-Don, Russian Federation)

⁴Rhein-Main University of Applied Sciences (Wiesbaden, Germany)

Mechanics

Funding information: The work (first author) was supported by the Government of the Russian Federation (contract No. 075-15-2019-1928). The third author carried out work within the framework of the State Assignment of the Southern Scientific Center of the Russian Academy of Sciences (State Reg. Project AAA-A-A16-116012610052-3).

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Introduction. In the production of sensor and measuring systems, small household appliances, cell phones, and wireless sensor systems, powerful energy sources are not needed to monitor and diagnose the technical condition of objects. In this case, the prerequisites are the mobility and volatility of the above devices.

Piezoelectric materials directly convert electrical energy into mechanical energy and vice versa. This property allows them to be widely used in science and technology. These materials are used in ultrasonic emitters of elastic and acoustic waves, receivers of such waves, devices for suppressing vibrations of machine elements and structures, etc. Recently, another field of application of piezoelectrics has been rapidly developing — energy collection and storage devices. In this case, piezoelectric materials are part of piezoelectric energy generators (PEG). PEG are placed on elements of machines or structures that vibrate intensively, which are in the zone of elastic wave propagation or are exposed to variable pressure. The main types of these devices have a bimorph or stack multilayer structure and experience bending or longitudinal deformations, respectively. Low-power sources of electric current are created on the basis of PEG. They include autonomous power sources (e.g., for damage monitoring devices in hard-to-reach places of pipeline structures, etc.). An overview of such devices is available in [1–2]. One of the ways to design effective PEG is the use of piezoactive composites of various types of connectivity and heterogeneous materials based on piezoceramics, including porous one.

PEG, in whose design there are additional electromagnetic elements or permanent magnets, can fix or use the energy of an alternating magnetic field. One of the ways to solve this problem is the use of piezomagnetic materials in combination with piezoelectric ones. In this case, the alternating magnetic field causes the deformation of the piezomagnetic and the coupled piezoelectric, as a result, the latter generates electrical energy. There is a class of materials with ferromagnetic properties. Piezomagnetism is a phenomenon observed in some antiferromagnetic and ferromagnetic crystals. It is characterized by a linear relationship between the magnetic polarization of the system and mechanical deformation. In a piezomagnetic material, a spontaneous magnetic moment can be induced through applying mechanical stress, or deformation by applying a magnetic field. In studies of piezomagnetic materials [3–5], $CoFe_2O_4$ is very often considered. In [6–8], a composite based on $CoFe_2O_4$ and $BaTiO_3$ with piezoelectric and piezomagnetic properties is studied simultaneously.

Solutions to the problems of electroelasticity and magnetoelasticity are given in [9–11]. In [12], applied theories of vibrations of multilayer piezoelectric plates were developed considering the specifics of the distribution of electric potential over the structure thickness.

The problems on steady-state oscillations of an electro-magneto-elastic layer and a half-space under the action of harmonic loads are presented in [13, 14]. The prestressing is taken into account, as well as various electrical and magnetic conditions at the boundaries. The effect of these factors on the dispersion properties is investigated.

Earlier [15, 16], an applied theory was developed that considers the inhomogeneous distribution of the electric potential in the longitudinal direction, and the quadratic dependence on thickness. In the same papers, the stress-strain and electrical state of a hinged and cantilevered bimorph is investigated. In both cases, the applied theory showed good convergence with the finite element modeling results. The authors also developed an applied theory of bimorph

vibrations [17] consisting of an electroelastic and magnetoelastic layer. This approach is in good agreement with the finite element analysis results.

In this paper, the vibrations of the device are considered in the framework of a plane deformation. Based on the variational principle, an applied theory of bending vibrations of a two-layer piezoelectric bimorph is constructed. For steady-state vibrations, boundary conditions and a system of differential equations are obtained for four unknown functions (deflection, electric potential in the middle of the layer, magnetic potential in the middle of the layer and at the outer boundary), depending on the length of the bimorph. The influence of different percentage volume ratios of the bimorph composition on deflection, electric and magnetic potentials in certain positions, is investigated. The study results provide selecting the makeup of a composite piezomagnetoelectric material to achieve the most efficient operation of the device.

Materials and Methods. A plate consisting of two identical piezoelectric layers is considered. It performs steady-state transverse vibrations within a plane deformation. Each layer is a 2–2 connectivity composite consisting of alternating piezoelectric and piezomagnetic layers (Fig. 1).

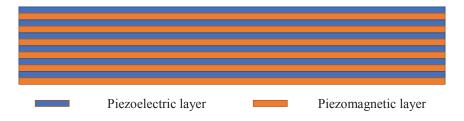


Fig. 1. 2-2 composite structure

Effective properties of such a composite were found in [8]. Large surfaces of the layers are electrodated, and the layers themselves are polarized in thickness. The bimorph is hinged at the edges, all surfaces are free from mechanical stresses. The upper and lower boundaries of the plate are affected by magnetic flux B_0 , while at the boundary between the layers, the magnetic potential is considered to be zero. The electrical potential is zero at all electrodes. The side surfaces are considered to be insulated from magnetic and electric fields.

The equations for describing the vibrations of a composite with effective properties, connectivity of mechanical, electric and magnetic fields, have the form [18]:

$$\nabla \cdot \sigma + \rho f = \rho \ddot{u}, \quad \nabla \cdot D = \sigma_{\Omega}, \quad \nabla \cdot B = 0,$$

$$\sigma = c : \varepsilon - e^{T} \cdot E - h^{T} \cdot H,$$

$$D = e : \varepsilon + \kappa \cdot E + \alpha \cdot H,$$

$$B = h : \varepsilon + \alpha^{T} \cdot E + \mu \cdot H,$$

$$\varepsilon = \frac{1}{2} \Big(\nabla u + (\nabla u)^{T} \Big), \quad E = -\nabla \phi, \quad H = -\nabla \xi.$$
(1)

Here, σ and ε — mechanical stress and strain tensors, D and E — vectors of electric induction and electric field strength, B and B — vectors of magnetic induction and magnetic field strength, D — material density, D — elastic moduli tensor, D — piezoelectric moduli tensor, D — piezomagnetic moduli tensor, D — dielectric permittivity tensor, D — magnetoelectric moduli tensor, D — magnetic permeability tensor, D — mass force density vector, D0 — electric charge volume density, D1 — displacement vector, D2 and D3 — electrical and magnetic potentials.

The boundary conditions are determined for the mechanical, electric and magnetic fields, respectively. For the first case, we note the absence of mechanical stresses at the bimorph boundary:

$$\sigma_{ij} \cdot n_j \Big|_{S} = 0, \quad i, j = 1.3.$$

The biomorph is hinged at the ends (Fig. 2):

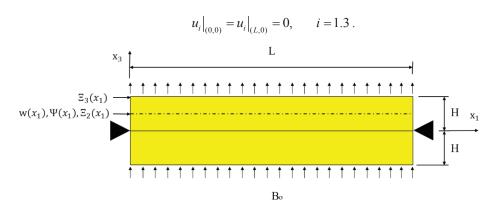


Fig. 2. Geometry and boundary conditions of bimorph with composite piezomagnetoelectric layers

Next, we formulate the electrical boundary conditions. Electrical potential on the internal and external electrode, respectively:

$$\varphi\big|_{\mathbf{r}_2=0} = V_0, \qquad \varphi\big|_{\mathbf{r}_2=H} = V_2.$$

We indicate the magnetic boundary conditions. Magnetic potential at the inner boundary:

$$\xi\big|_{x_0=0}=M_0.$$

Magnetic flux B_0 affects the upper and lower boundaries of the plate:

$$H\big|_{Y_0=+H}=B_0$$
.

We use the variational equation for steady-state vibrations [10]. It generalizes Hamilton's principle in the electroelasticity theory taking into account magnetic components. For the case of plane deformation in the absence of surface loads and in the presence of magnetic flux:

$$\iint_{S} \delta \tilde{H} dS - \rho \omega^{2} \iint_{S} u_{i} \delta u_{i} dS + \iint_{S} (p_{i} \delta u_{i} + \sigma_{0} \delta \varphi + B_{0} \delta \xi) dS = 0, \qquad (2)$$

where $\delta \tilde{H} = \sigma_{ij} \delta \epsilon_{ij} - D_i \delta E_i - B_i \delta H_i$.

To construct an applied theory of vibrations, we will accept Kirchhoff's hypotheses. In accordance with them, the distribution of displacements along the thickness has the form:

$$u_1(x_1, x_3) = -x_3 w_1, u_3(x_1, x_3) = w(x_1)$$
 (3)

In particular, the single normal hypothesis is accepted for a mechanical field. Next, we consider a problem in which the value of the electric potential on the electrodes can be zero, so its distribution is not described by a linear function. Taking into account the possible inhomogeneity in the length of the element associated with the influence of boundary conditions at the ends of the bimorph, its thickness distribution is assumed to be quadratic:

$$\varphi(x_1, \tilde{x}_3) = V_0(x_1) \frac{\tilde{x}_3}{H} \left(\frac{2\tilde{x}_3}{H} - 1 \right) + V_1(x_1) \left(1 - \frac{4\tilde{x}_3^2}{H^2} \right) + V_2(x_1) \frac{\tilde{x}_3}{H} \left(\frac{2\tilde{x}_3}{H} + 1 \right). \tag{4}$$

Here, $\tilde{x}_3 = x_3 - h/2$. Functions V_0 , V_1 and V_2 are responsible for the value of the electric potential at the inner electrode, in the middle of the layer, and at the outer electrode, respectively. To satisfy the conditions of the problem, we take these functions in the following form (see Fig. 2):

$$V_0(x_1) = V_0 = const, \ V_1(x_1) = \Phi(x_1), \ V_2(x_1) = V_2 = const.$$

Here, function $\Phi(x_1)$ is unknown.

We represent a quadratic distribution of the magnetic potential over the thickness of each layer. The distribution along the length is heterogeneous, at the inner boundary of the layers, its value is assumed to be zero:

$$\xi(x_1, \tilde{x}_3) = M_0(x_1) \frac{\tilde{x}_3}{H} \left(\frac{2\tilde{x}_3}{H} - 1 \right) + M_1(x_1) \left(1 - \frac{4\tilde{x}_3^2}{H^2} \right) + M_2(x_1) \frac{\tilde{x}_3}{H} \left(\frac{2\tilde{x}_3}{H} + 1 \right). \tag{5}$$

Here, $\tilde{x}_3 = x_3 - h/2$. Functions M_0 , M_1 and M_2 are responsible for the value of the magnetic potential at the inner boundary, in the middle of the layer, and at the outer boundary, respectively, and are taken as follows (Fig. 2):

$$M_0(x_1) = M_0 = const$$
, $M_1(x_1) = \Xi_2(x_1)$, $M_2(x_1) = \Xi_3(x_1)$.

Here, functions $\Xi_2(x_1)$ and $\Xi_3(x_1)$ are unknown.

We substitute relations (3)–(5) into equation (2) and integrate it by the bimorph thickness, and then we equate the coefficients with independent variations δw , $\delta \Phi$, $\delta \Xi_2$ and $\delta \Xi_3$ to zero. Thus, we obtain a system of four differential equations (6) from four unknown functions depending on x_1 (then, we omit the subscript), and five boundary conditions (7).

$$\begin{split} &\frac{16\tilde{\epsilon}_{33}}{3H}V_0 + \frac{16\tilde{\epsilon}_{33}}{3H}V_2 - \frac{32\tilde{\epsilon}_{33}}{3H}\Phi(x) - \frac{32\tilde{\alpha}_{33}}{3H}\Xi_2(x) + \frac{16\tilde{\alpha}_{33}}{3H}\Xi_3(x) - \frac{16\epsilon_{11}H}{15}\frac{d^2}{dx^2}\Phi(x) - \\ &- \frac{16\alpha_{11}H}{15}\frac{d^2}{dx^2}\Xi_2(x) - \frac{2\alpha_{11}H}{15}\frac{d^2}{dx^2}\Xi_3(x) - \frac{4\tilde{\epsilon}_{31}H}{3H}\frac{d^2}{2}w(x) + \frac{16\tilde{\alpha}_{33}}{3H}M_0 = 0, \\ &\frac{16\tilde{\alpha}_{33}}{3H}V_0 + \frac{16\tilde{\alpha}_{33}}{3H}V_2 - \frac{32\tilde{\alpha}_{33}}{3H}\Phi(x) - \frac{32\tilde{\mu}_{33}}{3H}\Xi_2(x) + \frac{16\tilde{\mu}_{33}}{3H}\Xi_3(x) - \frac{16\alpha_{11}H}{15}\frac{d^2}{dx^2}\Phi(x) - \\ &- \frac{16\mu_{11}H}{15}\frac{d^2}{dx^2}\Xi_2(x) - \frac{2\mu_{11}H}{15}\frac{d^2}{dx^2}\Xi_3(x) - \frac{4\tilde{h}_{31}H}{3H}\frac{d^2}{dx^2}w(x) + \frac{16\tilde{\mu}_{33}}{3H}W_0 = 0, \\ &- \frac{2\tilde{\alpha}_{33}}{3H}V_0 - \frac{14\tilde{\alpha}_{33}}{3H}V_2 + \frac{16\tilde{\alpha}_{33}}{3H}\Phi(x) + \frac{16\tilde{\mu}_{33}}{3H}\Xi_2(x) - \frac{14\tilde{\mu}_{33}}{3H}\Xi_3(x) - \frac{2\alpha_{11}H}{15}\frac{d^2}{dx^2}\Phi(x) - \\ &- \frac{2\mu_{11}H}{15}\frac{d^2}{dx^2}\Xi_2(x) - \frac{4\mu_{11}H}{15}\frac{d^2}{dx^2}\Xi_3(x) + \frac{5\tilde{h}_{31}H}{3}\frac{d^2}{dx^2}w(x) - 2B_0 - \frac{2\tilde{\mu}_{33}}{3H}M_0 = 0, \\ &\frac{4\tilde{\epsilon}_{31}H}{3}\frac{d^2}{dx^2}\Phi(x) + \frac{4\tilde{h}_{31}H}{3}\frac{d^2}{dx^2}\Xi_2(x) - \frac{5\tilde{h}_{31}H}{3}\frac{d^2}{dx^2}\Xi_3(x) - 2p_3H - \\ &- \frac{2\rho\omega^2H^3}{3}\frac{d^2}{dx^2}w(x) + 2\omega^2\rho_W(x)H + \frac{2H^3\tilde{\epsilon}_{11}}{3}\frac{d^4}{dx^4}w(x) = 0. \end{split}$$

Here, the following designations were introduced: $\tilde{c}_{11} = c_{11} - c_{13}^2 / c_{33}$, $\tilde{e}_{31} = e_{31} - c_{13}e_{33}/c_{33}$, $\tilde{h}_{31} = h_{31} - c_{13}h_{33}/c_{33}$, $\tilde{\alpha}_{33} = -\alpha_{33} - e_{33}h_{33}/c_{33}$, $\tilde{\epsilon}_{33} = -\epsilon_{33} - e_{33}^2/c_{33}$. They occurred after satisfying condition $\sigma_{33} = 0$ and exclusion of ε_{33} .

Research Results. The results of the bimorph calculation according to the proposed theory are compared to the finite element calculation in the low-frequency region for the volume ratio of the piezoelectric and piezomagnetic components $80 \% BaTiO_3$ and $20 \% CoFe_2O_4$. The comparison has shown that the error in finding the characteristics of the mechanical and magnetic fields is less than 1%. When determining the electric field in the middle part of the plate, the difference was about 5%. Describing the situation in the vicinity of the support points, it should be noted that the size of the neighborhood along the longitudinal coordinate is approximately equal to the bimorph thickness. Here, when determining the electric field, a difference of 20% is recorded.

volume ratio of $BaTiO_3$ and $CoFe_2O_4$ in the composite is to determine its effective properties. Tables 1 and 2 present these properties found from the results of study [8].

Table 1

The first step in studying the vibrations of a two-layer piezomagnetoelectric bimorph with a change in the

Table 1 Material constants (elastic modules, dielectric and magnetic permittivity) for different $BaTiO_3$ volume fraction

Volume	Elastic modules					Dielectric p	permittivity	Magnetic permittivity	
fraction			GPa			10^{-9} f/m		$10^{-4} \text{ N s}^2/\text{C}^2$	
$BaTiO_3$ (%)	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	к ₁₁	к ₃₃	μ_{11}	μ_{33}
0	286.0	173.0	170.0	269.5	45.30	0.080	0.093	5.900	1.570
10	270.9	160.4	154.9	260.0	45.07	1.469	0.073	5.315	0.632
20	256.6	148.5	142.6	250.2	44.84	2.815	0.098	4.730	0.396
30	242.8	137.2	131.3	240.8	44.61	4.063	0.122	4.145	0.285
40	229.9	126.8	120.9	231.9	44.38	5.287	0.147	3.560	0.223
50	217.6	116.9	111.0	224.0	44.15	6.413	0.171	2.975	0.186
60	206.7	108.1	102.1	215.6	43.92	7.490	0.220	2.390	0.155
70	195.9	99.7	93.8	208.2	43.69	8.517	0.294	1.805	0.136
80	186.0	92.3	85.9	201.3	43.46	9.448	0.441	1.220	0.120
90	176.6	85.4	78.9	193.9	43.23	10.353	0.857	0.635	0.110
100	166.0	77.0	78.0	162.0	43.00	11.200	12.600	0.050	0.100

Table 2 Material constants (piezoelectric, piezomagnetic and magnetoelectric modules) for different $BaTiO_3$ volume fraction

Volume	Piezoelectric modules			Piezon	nagnetic mo	dules	Magnetoelectric modules	
fraction	C/m ²				N/A m		10 ⁻⁸ Ns/VC	10 ⁻¹¹ Ns/VC
$BaTiO_3$ (%)	e_{31}	$e_{_{33}}$	e_{15}	h_{31}	h_{33}	h_{15}	α_{11}	α_{33}
0	0	0	0	580.3	-699.7	550	0	0
10	-0.006	0.029	1.16	223.6	-244.1	495	-1.33	1.97
20	-0.013	0.059	2.32	130.0	-132.3	440	-2.35	2.36
30	-0.019	0.088	3.48	86.7	-79.8	385	-3.07	2.48
40	-0.025	0.132	4.64	61.6	-52.5	330	-3.48	2.50
50	-0.031	0.176	5.80	43.3	-34.2	275	-3.62	2.47
60	-0.038	0.220	6.96	29.7	-22.8	220	-3.45	2.43
70	-0.040	0.352	8.12	20.5	-13.7	165	-3.00	2.36
80	-0.060	0.571	9.28	13.7	-9.1	110	-2.27	2.29
90	-0.263	1.187	10.44	4.6	-4.6	55	-1.28	2.16
100	-4.400	18.600	11.60	0	0	0	0	0

The bimorph vibrations were excited by a magnetic flux applied to the upper and lower faces (Fig. 2), which varied according to the harmonic law with an amplitude of $B_0 = 5 \times 10^{-5}$ Wb and a frequency of 10 kHz.

Figure 3 shows the deflection in the middle of the layer depending on volume fraction of $BaTiO_3$. It can be seen from the graph that the deflection in the position having coordinates $x_1 = L/2$, $x_3 = H/2$, is zero if the bimorph consists only of a piezoelectric $BaTiO_3$. The deflection of the bimorph reaches the greatest value if it contains only piezoelectric magnet $CoFe_2O_4$. The deflection almost linearly depends on the volume ratio of the components of piezoactive materials.

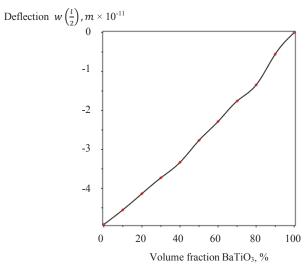


Fig. 3. Deflection $w(x_1)$ in the middle of the layer for different $BaTiO_3$ volume fraction

Based on the data in Figure 4, it can be concluded that the electric potential in the middle of the layer varies nonlinearly with a change in the volume ratio of the composition of bimorph piezoactive materials. If the bimorph consists only of $BaTiO_3$ or $CoFe_2O_4$, then the electric potential at the point (L/2, H/2) is zero and reaches the highest value at 35 % $BaTiO_3$ in the bimorph.

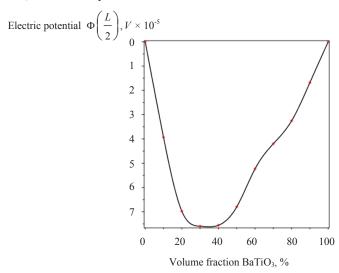


Fig. 4. Electric potential $\Psi(x_1)$ for different $BaTiO_3$ volume fraction

The analysis of Figures 5 and 6 allows us to conclude that the magnetic potential in the middle of the layer $\Xi_2(L/2)$ and at the outer boundary $\Xi_3(L/2)$ increases almost linearly with an increase in the volume of $BaTiO_3$ in the bimorph.

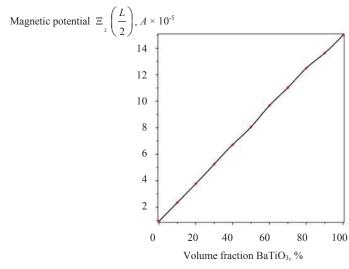


Fig. 5. Magnetic potential $\Xi_2(L/2)$ for different $BaTiO_3$ volume fraction

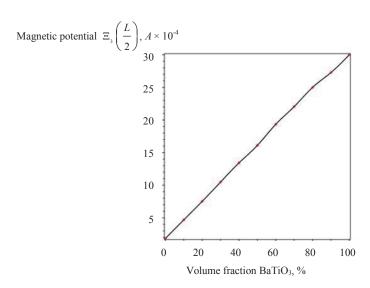


Fig. 6. Magnetic potential $\Xi_3(L/2)$ for different $BaTiO_3$ volume fraction

Discussion and Conclusions. An applied theory is proposed for calculating transverse vibrations of a bimorph made of two layers of a composite based on $CoFe_2O_4$ and $BaTiO_3$ with both piezoelectric and piezomagnetic properties, in an alternating magnetic field. Such a design can serve as a model of a piezoelectric generator of a device for collecting and storing energy under the action of an external magnetic field. In the low-frequency region (below the natural frequency of the first bending mode), calculations of the stress-strain state of the bimorph, the distribution of electric and magnetic fields, are carried out. The dependence of the deflection, electric and magnetic potentials on the volume ratio of the bimorph composition is investigated. In further work, it is assumed to determine the output potential and power of an electric current excited by an alternating magnetic field. The purpose of these surveys will be to collect electrical energy.

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Received 23.12.2021 Revised 21.01.2022 Accepted 24.01.2022

About the Authors

Soloviev, Arkadiy N., Head of the Theoretical and Applied Mechanics Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344003, RF), Dr.Sci. (Phys.-Math.), professor, ResearcherID, ScopusID, ORCID, solovievarc@gmail.com.

Do Thanh Binh, postgraduate student of the Theoretical and Applied Mechanics Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344003, RF), <u>ResearcherID</u>, <u>ScopusID</u>, <u>ORCID</u>, dothanhbinh@mail.ru.

Chebanenko, Valerii A., Senior Research Scholar, Federal Research Centre the Southern Scientific Centre of the Russian Academy of Sciences (41, Chekhov St., Rostov-on-Don, 344010, RF), Cand.Sci. (Phys.-Math.), senior research scholar, ResearcherID, ScopusID, ORCID, valera.chebanenko@yandex.ru.

Lesnyak, Olga N., associate professor of the Theoretical and Applied Mechanics Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344003, RF), ORCID, lesniak.olga@yandex.ru

Kirillova, Evgeniya V., Rhein-Main University of Applied Sciences (Wiesbaden, Germany (18, Kurt-Schumacher-Ring, Wiesbaden 65197, Germany), Cand.Sci. (Phys.-Math.), professor, <u>ScopusID</u>, <u>ORCID</u>, <u>Evgenia.Kirillova@hs-rm.de</u>

Claimed contributorship

A. N. Soloviev: academic advising; task formulation; analysis of the research results. Do Thanh Binh: derivation of equations; calculation program development; computational analysis; analysis of the research results; text preparation. V. A. Chebanenko: selection of research method; variational formulation of the problem; analysis of the research results. O. N. Lesnyak: text preparation; discussion of the results. E. V. Kirillova: analysis of the research results.

All authors have read and approved the final manuscript.